Let M be the maximum of ten independent samples of a Uniform$(0, \theta)$ random variable. What is the smallest value of t for which (M, tM) is a 95%-confidence interval for θ?

Solution: We are looking for the smallest t such that $\mathrm{P}(M \leq \theta \leq tM) \geq 0.95$. As $\theta \geq M$ with probability one, the event $M \leq \theta \leq tM$ is the same as $\theta \leq tM$, or equivalently $M \geq \theta/t$. As the samples $X_1, \dots, X_{10}$ are independent,

$$\mathrm{P}(M \geq \theta/t) = 1 - \mathrm{P}(M < \theta/t) = 1 - \mathrm{P}(X_1 < \theta/t) \cdots \mathrm{P}(X_{10} < \theta/t) = 1 - 1/t^{10},$$

so t should be chosen as small as possible so that $1 - 1/t^{10} \geq 0.95$, or $t \geq (1/0.05)^{1/10} \approx 1.349$. Therefore $t \approx 1.349$.