The PM10 pollutant index was estimated to 50 μg/m^3 on Monday and 35 μg/m^3 on Tuesday. Assuming each day's readings are derived from 9 independent samples of a normal random variable with standard deviation $\sigma = 6$ μg/m^3, find the p-value for the alternative hypothesis "the PM10 density has decreased".

Solution: We apply the test for comparing normal means. Let μ_M and μ_T be the actual pollutant densities on Monday and Tuesday. The difference $\overline{M}-\overline{T}$ between Monday's and Tuesday's estimates is a Normal$(\mu_M - \mu_T, \sqrt{2\sigma^2/n})$ random variable with standard deviation $\sqrt{2\sigma^2/n} = \sqrt{2 \cdot 6^2/9} = \sqrt{8}$. The test is one-sided, so it should accept the alternative hyphothesis when $\overline{M} - \overline{T} \geq t$ for a suitable threshold t. Under the null hypothesis $\overline{M} - \overline{T}$ is a Normal$(0, \sqrt{8})$ random variable so the p-value is

$$\mathrm{P}(\mathrm{Normal}(0, \sqrt{8}) \geq 50 - 35) \approx \mathrm{P}(\mathrm{Normal}(0, 1) \geq 5.30) \approx 5 \cdot 10^{-8}$$

so the p-value is very close to zero and the confidence in the alternative hypothesis is very high.